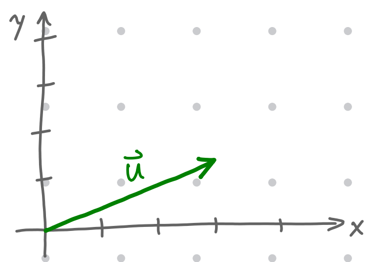
MATH 126 A/B — 3 November 2025

VECTORS: indicate length and direction

Example:



Component notation:

$$\vec{u} = \langle 3, 1 \rangle$$

3 indicates Δx

1 indicates Δy

Note: angle brackets distinguish vectors from points

LENGTH (MAGNITUDE) OF A VECTOR:

2D: If $\vec{u} = \langle a, b \rangle$, then $|\vec{u}| = \sqrt{a^2 + b^2}$

3D: If $\vec{u} = \langle a, b, c \rangle$, then $|\vec{u}| = \sqrt{a^2 + b^2 + c^2}$

SPECIAL VECTORS:

standard basis vectors

2D: $\vec{\imath} = \langle 1, 0 \rangle$, $\vec{\jmath} = \langle 0, 1 \rangle$

3D: $\vec{\imath} = \langle 1, 0, 0 \rangle$, $\vec{\jmath} = \langle 0, 1, 0 \rangle$, $\vec{k} = \langle 0, 0, 1 \rangle$

These are all unit vectors, meaning that their lengths are exactly one!

UNIT VECTOR IN A GIVEN DIRECTION:

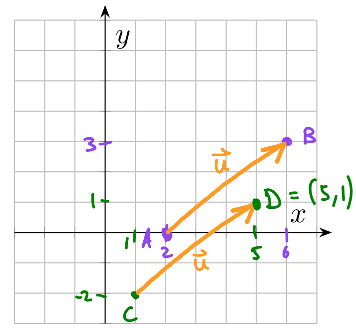
Given a nonzero vector $\vec{u}$, a unit vector in the direction of $\vec{u}$ is found by dividing $\vec{u}$ by its magnitude:

$\dfrac{1}{|\vec{u}|}\vec{u}$ is a unit vector in the direction of $\vec{u}$.

Vectors

1. Let $A = (2, 0)$ and $B = (6, 3)$.

(a) Draw the vector from A to B (which we will call $\mathbf{u} = \overrightarrow{AB}$).



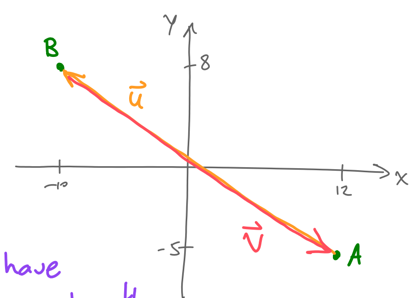
$\vec{u} = \langle \underbrace{6-2}_{\Delta x}, \underbrace{3-0}_{\Delta y} \rangle = \langle 4, 3 \rangle$

(b) Suppose $C = (1, -2)$ and the vector from C to D has the same length and direction as the vector $\mathbf{u}$. Find the coordinates of point D.

☞ Vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are said to be *equal*.

$D = C + \vec{u} = (1, -2) + \langle 4, 3 \rangle = (5, 1)$

2. Now suppose $A = (12, -5)$ and $B = (-10, 8)$.



(a) Suppose $\mathbf{u}$ is the vector from A to B. If $\mathbf{u} = \langle x, y \rangle$, find x and y.

$\vec{u} = \langle -10 - 12, 8 - (-5) \rangle = \langle -22, 13 \rangle$

(b) Suppose $\mathbf{v}$ is the vector from B to A. If $\mathbf{v} = \langle x, y \rangle$, find x and y.

$\vec{v} = \langle 22, -13 \rangle$

(c) Are $\mathbf{u}$ and $\mathbf{v}$ equal? Why or why not? Not equal, but they have opposite directions and same length.

$\vec{u} = -1\vec{v}$ multiply each component of $\vec{v}$ by -1

3. Consider the vectors $\mathbf{u} = \langle 4, 1 \rangle$ and $\mathbf{v} = \langle 1, 2 \rangle$.

(a) Compute the <u>*sum*</u> $\mathbf{u} + \mathbf{v}$.

To add vectors, add their components!

$\vec{u} + \vec{v} = \langle 4, 1 \rangle + \langle 1, 2 \rangle = \langle 4+1, 1+2 \rangle = \langle 5, 3 \rangle$

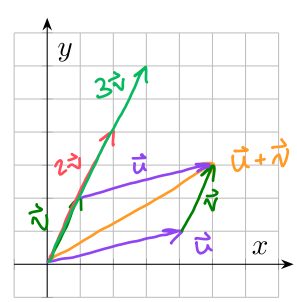
☞ Make your best guess as to HOW to add two vectors.

(b) Sketch $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u} + \mathbf{v}$ on the grid below (emanating from the origin). Geometrically, how are these three vectors related to each other?

(c) Use your sketch to explain why $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.

$\vec{u} + \vec{v}$ is a diagonal of the parallelogram formed by $\vec{u}$ and $\vec{v}$.

$2\vec{v} = 2\langle 1, 2 \rangle$
$= \langle 2 \cdot 1, 2 \cdot 2 \rangle$
$= \langle 2, 4 \rangle$



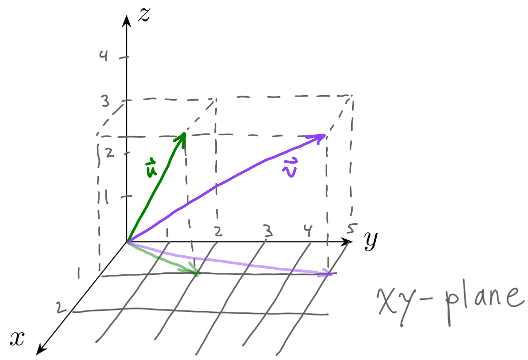
(d) How would you interpret $2\mathbf{v}$ and $3\mathbf{v}$? Sketch these vectors on the grid above.

Multiply a vector by a number (scalar): multiply each component of the vector by the scalar

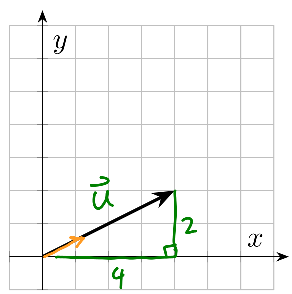
Example: $c\langle a, b \rangle = \langle ca, cb \rangle$

4. *Now let's move up to three dimensions.* To the best of your ability, sketch the vectors $\mathbf{u} = \langle 1, 2, 3\rangle$ and $\mathbf{v} = \langle 1, 5, 3\rangle$ below.

☞ First draw the "shadow" in the xy-plane.



5. (a) The vector $\mathbf{u} = \langle 4, 2\rangle$ is drawn below. What is the length of $\mathbf{u}$? How did you compute its length?



length of $\vec{u}$:

$$\sqrt{4^2 + 2^2} = \sqrt{20}$$
$$= 2\sqrt{5}$$

(b) **Group chat:** How do you compute the length of the 2-dimensional vector $\langle a, b\rangle$?

If $\vec{u} = \langle a, b\rangle$, then $|\vec{u}| = \sqrt{a^2 + b^2}$.

(c) **Group chat:** How do you compute the length of the 3-dimensional vector $\langle a, b, c\rangle$?

If $\vec{u} = \langle a, b, c\rangle$, then $|\vec{u}| = \sqrt{a^2 + b^2 + c^2}$.

6. Let $\mathbf{u} = \langle 4, 2\rangle$ and $\mathbf{v} = \langle 4, 2, -7\rangle$.

(a) Write the vectors $\mathbf{u}$ and $\mathbf{v}$ in terms of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

$\vec{u} = 4\vec{i} + 2\vec{j}$

$\vec{v} = 4\vec{i} + 2\vec{j} - 7\vec{k}$

(b) Find a unit vector in the same direction as $\mathbf{u}$. Find a unit vector in the same direction as $\mathbf{v}$.

To find a unit vector in the direction of $\vec{u}$, multiply $\vec{u}$ by $\frac{1}{|\vec{u}|}$.

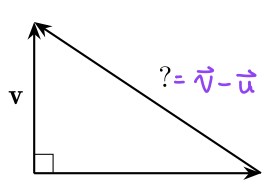
$$\frac{1}{|\vec{u}|}\vec{u} = \frac{1}{\sqrt{4^2+2^2}}\vec{u} = \frac{1}{\sqrt{20}}\langle 4, 2\rangle = \left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$$

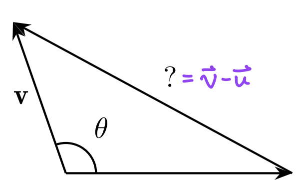
Check that this is a unit vector $\longrightarrow$ $\left|\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right\rangle\right| = \sqrt{\left(\frac{2}{\sqrt{5}}\right)^2 + \left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{\frac{4}{5} + \frac{1}{5}} = \sqrt{1} = 1$

$$\frac{1}{|\vec{v}|}\vec{v} = \frac{1}{\sqrt{4^2+2^2+7^2}}\vec{v} = \frac{1}{\sqrt{69}}\langle 4, 2, -7\rangle = \left\langle \frac{4}{\sqrt{69}}, \frac{2}{\sqrt{69}}, \frac{-7}{\sqrt{69}} \right\rangle$$

7. Given vectors $\mathbf{u}$ and $\mathbf{v}$ in each diagram, find the unknown vector. How are $|\mathbf{u}|$, $|\mathbf{v}|$, and $|?|$ related to the angle?

$20 + 49 = .$





$|\vec{v}|^2 + |\vec{u}|^2 = |\vec{v} - \vec{u}|^2$

How are the lengths of $\vec{u}$, $\vec{v}$, and $\vec{v}-\vec{u}$ related? Does θ play a role?